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THESIS

PREDICTING HAIL SIZE USING MODEL VERTICAL VELOCITIES

by

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March 2008

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A simple test hail growth model is created in order to compare hailstone sizes from model vertical velocities and calculated updrafts from a simple cloud model using forecasted soundings. The models used MM5 model data coinciding with severe hail events collected from the Central and Southern Plains from March to May 2006 and 2007. In the test model, four different starting embryo sizes were interjected into four separate hail growth modes: dry growth and wet growth using model vertical velocities and dry and wet growths using calculated updrafts. These embryos were placed at four different beginning vertical levels resulting in 64 possible ending hailstone sizes. Examination of the 804 hail events revealed the potential usefulness of model vertical velocities in generating severe hailstones. In particular, using dry growth, the model vertical velocities produced 727 severe hailstones compare to 661 produced by dry growth using the thermodynamically calculated updraft. Model vertical velocities also proved more accurate than updrafts, resulting in an average error of 0.417 compare to 0.788 under dry growth conditions. Calculated updrafts were still required to generate the large severe hail that model vertical velocities could not produce.

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PREDICTING HAIL SIZE USING MODEL VERTICAL VELOCITIES

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Submitted in partial fulfillment of the
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I. INTRODUCTION

Hail forms into several different shapes and structures by different growth methods and starting embryos. Even during the life span of a single thunderstorm, hail can change shape and structure from the beginning to the end (Knight and Knight 1970). Hail also results in severe damage to structures, crops, and especially aircraft, costing millions of dollars of damage. Too often hail size is not known until someone observes it hitting the ground, limiting necessary lead time in order for people or companies to protect their property or equipment. By this time it is generally too late to take preventative measures, such as sheltering sensitive equipment. Doppler radar provides some capability for estimating hail occurrence and to a lesser extent hail size, but once again only after thunderstorms develop. In order to maximize lead time, an accurate hail forecast is required many hours in advance of thunderstorm development. Although hail forecast products exist currently, they lack the necessary studies to determine their accuracy. Plus they are based primary on observed upper air soundings. Obtaining upper air soundings in proximity to convective activity is a challenge due to that high spatial and temporal variability of convection. Furthermore, methods or models that calculate updrafts via the parcel theory typically overestimate the updraft magnitude and lead to unrealistic hail sizes. Therefore, the model proposed in this thesis will examine the possibility of substituting observed soundings with forecast soundings from a non-hydrostatic model, the MM5. This model, along with others, contains the complex cloud microphysics that was tested years ago in 2-D or 3-D cloud and hail models. In addition, model vertical velocities from a non-hydrostatic model will include a convective component that other hydrostatic models would not. Thus, this research will also examine the hail growth differences between using model vertical velocity and the updraft calculated from a simple 1-D cloud model in order to determine if these model vertical velocities can produce accurate hailstone sizes. The area of concentration for these comparisons will focus on the Central and Southern Plains during the 2006 and 2007 spring seasons (Mar to May) using Air Force Weather Agency’s 15 km MM5.

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II. BACKGROUND

A. EARLY DAYS

Hail occurs frequently over the Central and Southern Plains (Figure 1) every spring and summer, yet forecasters continue to struggle to narrow the forecast of the size and coverage of the hailstones. Forecasters understand the simple philosophy behind formation and growth of hail; however operational models used currently cannot resolve the multitude of microphysical interactions involved in growing a hailstone to severe size. Researchers and forecasters have understood the basic ingredients necessary for formation (strong updraft, moisture, cold temperatures aloft, and embryo), and to a lesser extent the growth of hail, for the past 60 years. How these ingredients interact with each other and vary within the environment is extremely complicated to model or is simply unknown at this time. Furthermore, until recently, computational requirements prevented these multi-dimensional hail models from being used operationally. Therefore the early researchers focused on a single component or ingredient of hail formation or growth that was observable in nature or in a lab in order to gain a more complete understanding of how it contributes.

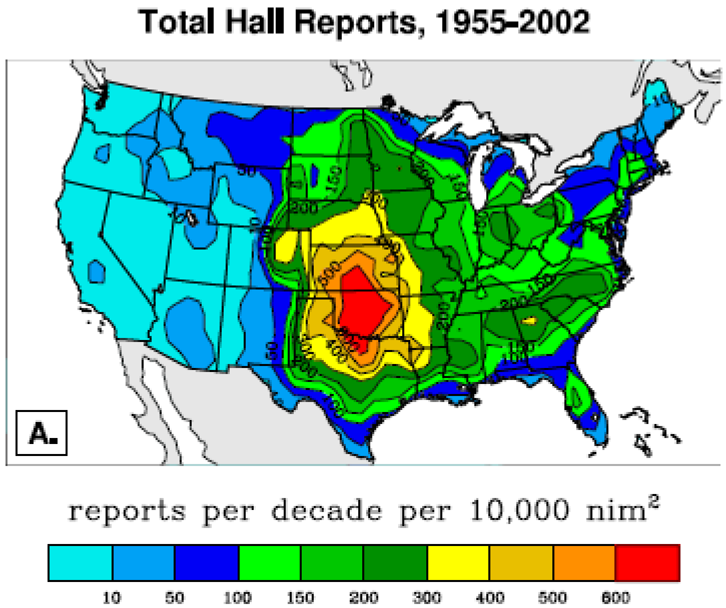

Figure 1. April Distribution of Severe Hail Events (From Schaefer et al., 2002)

In the beginning, updraft or vertical velocity was the primary focus. Researchers could only calculate updrafts from collected hail or used experimental updrafts based on the simple relationship of density and hailstone size to estimate upward velocity (Humphreys 1928 and Bilham and Relf 1937). Fawbush and Miller (1953) used their experimental results, along with the results from Grimminger (1933) on variations in turbulence, to construct a graph relating base and altitude of a positive triangle from an upper-air sounding to a forecasted hailstone diameter. This positive triangle is formed by lifting a parcel from the convective condensation level (CCL) along the saturated adiabat to the level of hail formation (-5 ˚C), and following that level over to where it intersects the environmental sounding. In the process of studying the hailstones and their thermodynamic properties, they discovered two other key ingredients in hail formation and growth. Moisture content and temperature lapse rates played a significant role in final hailstone size. This was indicated in the construction of the positive triangle from the upper-air sounding in their paper. Results were encouraging, however to achieve a high skill score, this method relied on the forecaster's skill in predicting the future upper air structure. Plus, they derived the data from upper-air soundings representative of a hailstone environment. They did not compare these to non-hailstone environmental

soundings. These discrepancies both contributed to the errors encountered when they verified the method; however the simplicity of this method allowed for continuing operational usage for nearly the next 40 years.

Foster and Bates (1956) also pursued the updraft approach to hail formation and growth. Their approach was more physical than that of Fawbush and Miller (1953), and based on three premises. The three premises were: updraft velocity equals the hailstone's terminal velocity (will accelerate downward until the aerodynamic drag force is just equal to its weight); vertical velocity is derived from the buoyancy force and velocity; hailstone size is calculated from a positive area on a thermodynamic diagram. They used these premises to arrive at an equation for updraft speed at the level of hail formation. This is essentially what Fawbush and Miller (1953) accomplished since the equation

$$W_h = (\frac{g}{T_m}\Delta T_h H)^{.5} \tag{1}$$

is related to the positive area analyzed on a skew-t. In this relationship, T_m is the mean temperature between the LCL and level of hail formation (H), and g is gravity. First they calculated various terminal velocities at various levels of the atmosphere for different hail diameters. Then, they solved Eq. (1) for $\Delta T_H H$. Finally by assigning values to H based on the terminal velocities already calculated, values for ΔT_H are obtained. Once again this is calculating hail size from the positive area on a thermodynamic chart, similar to Fawbush and Miller's technique. Foster and Bates then constructed a graph or hail size diagram relating hail size to ΔT_H and H for -10°C parcel temperatures at 400 mb. The forecaster could modify this graph if the -10˚C level did not reside at the 400 mb level. With adding these physical relationships between updraft and terminal velocity, Foster and Bates derived a method that showed a slight improvement (85% versus 83%) over the Fawbush-Miller method.

Two decades later Renick and Maxwell (1977) used the same idea to construct a graph relating maximum updraft and environmental temperature at maximum updraft to

hail size over Alberta, Canada. They used a cloud model to estimate the updraft magnitude and various other parameters in order to find highly correlated relationships between these parameters and hail size. During testing of this nomogram, they found that it forecasted the correct hail size 63% of the time and 80% of the time within one size category Renick and Maxwell (1977) continued the trend of deriving these products based on model rerun data from observed upper air soundings. However, they did begin to experiment with prognostic soundings, which were input to their cloud model in order to access the instability. They used this information in determining storm type, motion, and duration, region of storm development and onset time.

B. RECENT RESEARCH

More than 20 years ago, Pino and Moore (1991) introduced a new operational method for hail forecasting. It was based on the work of the previous mentioned researchers. By this time various researchers developed complex cloud and hail models (See Orville 1977 for list of models) which correlated several key physical components to hail growth. These models helped validate several assumption used by Fawbush and Miller (1953) and Foster and Bates (1956). They also included several other aspects of hail growth that were either ignored, unknown or simplified by the previous studies. Dry growth, wet growth, spongy growth, and melting were introduced and calculated using experimental data fields and complex computational algorithms. These models were excellent tools for researching, but impractical for operational use. Also other forecasters and researchers noticed the large errors and limitations of these two standard methods. Since these relied on observed soundings to predict hail size that sometimes occurred 12 hours out, hailstone size errors could be quite large (Leftwich 1984). Pino and Moore (1991) tried to solve that problem by producing a method that creates a forecast sounding from a 1200 UTC upper-air sounding. They applied the same relationship between the updraft (calculated from buoyancy) and the terminal velocity that Foster and Bates (1956) did. They instead used a vertical velocity from the Anthes (1977) 1-D cloud model which included entrainment rate, virtual mass coefficient and total liquid water. They calculated updraft by integrating from the level of free convection (LFC) and the CCL up

to the level of hail formation, or -10°C level in this case. Essentially this is the same positive area described by Fawbush and Miller (1953) and Foster and Bates (1956); however instead of combining the entire area into one and then computing updraft, they calculated the updraft at each level and then substituted back into the equation to compute a new hail radius at each level. It also allowed the forecaster to decide on whether to use the CCL or LFC based on the trigger mechanism for that day. They also included melting equations for hailstones which up to that point had not been included in an operational hail forecasting model of this magnitude. With the incorporated forecast soundings, validation results improved over results from the Fawbush and Miller (1953) technique. It showed that a forecast sounding could improve hail forecasting by predicting the necessary instability to allow for such strong updrafts.

In 2002 Brimelow (2002) introduced a new model called HAILCAST. This used a similar method to Pino and Moore (1991), however it combined a 1-D cloud model with a time dependent hail model. This time dependent hail model used Musil (1970) and Dennis and Musil (1973) for the hail growth equations and Rasmussen and Heymsfield (1987a, 1987b, and 1987c) for melting equations. Updrafts were once again calculated by using a 1-D cloud model similar to the one employed by Pino and Moore (1991). This method proved very successful with greater overall skill than the Renick and Maxwell (1977) method. It is also capable of distinguishing between non-severe and severe hail events. Jewel and Brimelow (2004) also tested HAILCAST in the U.S. Brimelow et al. (2006) examined HAILCAST once again over Alberta, Canada. In both studies, HAILCAST displayed skill in predicting the maximum hail diameter. This model also used an ensemble approach in addition to the single mode, by changing surface temperature and dewpoint for each model run. Changing the surface temperature and dewpoint greatly affects the atmosphere instability, which then affects the hailstone size. This ensemble mode produces a long list of possible hail sizes. Unlike the first two HAILCAST studies mentioned, the third one (2006) looked at the use of prognostic model soundings from the Global Environment Multiscale model. The results proved successful, indicated the likely improvement in hail size forecasting up to 12hrs out using model soundings or model vertical profiles.

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III. DATA AND THEORY

A. DATA

For this thesis, hail events were collected from the National Weather Service Storm Prediction Center (NWS SPC) events log from March to the end of May 2006 and 2007 over Nebraska, Kansas, Oklahoma, and Texas. The time of year was chosen due to the high occurrence of hail covering a large area over this region (Figure 2). Furthermore, the events were filtered to align them with a forecast hour, deleting reports more than 30 minutes before or after a forecast hour (00Z, 03Z, 06Z, 12Z, 15Z, 18Z or 21Z).

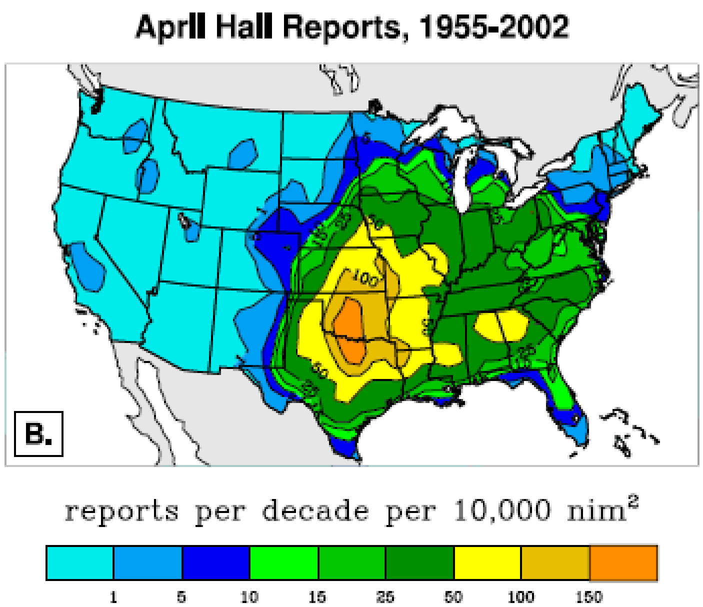

Figure 2. April Distribution of Severe Hail Events (From Schaefer et al., 2002)

For each hail report, a vertical sounding was retrieved from the AFWA MM5 model at the shortest forecast range. This sounding included the standard data found in atmosphere soundings, but also included omega at each level. In order to ensure consistency among the vertical profiles, events that fell within the window of 12-21Z, the 12Z model run was used. All other times, the 00Z run was used. For example, if a hail event occurred at 2130Z, the 12Z mode run would be used to create the 9-hr forecast

vertical profile. If the hail event occurred at 1500Z, then the 12Z model run time would be used to create the 3-hr forecast vertical profile. Once the vertical profiles are collected, only those for which the hail model could be run and verified were kept. This made the finally tally of 804 severe hail events (Figure 3).

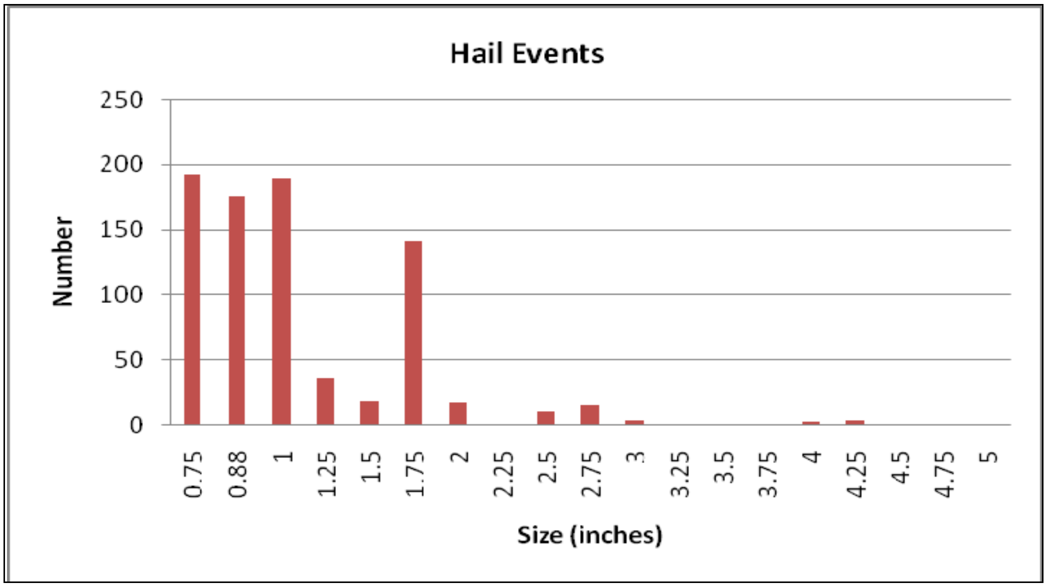

Figure 3. Hail event numbers from observed hail reports from March to May 2006 and 2007 over Kansas, Nebraska, Oklahoma, and Texas.

B. THEORY

Once this list of vertical profile data was created, they were injected into the test hail algorithm using MATLAB. The test hail algorithm is based on the Musil (1973) and Dennis and Musil (1973) hail models. The Musil method was chosen over other models because of its proven operational capability displayed in Brimelow's work with HAILCAST. In addition the model's physical equations were obtainable or calculable (by appropriate approximations) from the vertical profile profiles. Other more complicated models (see Orville 1977), such as Wisner et al., (1972), Ogura and Takahaski (1972), Takahsi (1976), and Xu (1983) are available; however the operational models of today include these cloud physics and dynamic equations. Since the goal of this study is to develop a simple algorithm to forecast hail size that does not depend on model specific microphysics and could be applied to a variety of models, use of the same

cloud equations would not provide an independent estimate. Furthermore the thesis assumes the clouds are around the mature stage, therefore initial cloud development is not required. Musil included growth rates for dry and wet growth, which are the primary methods of hail growth. He calculated both rates, with the one resulting in smallest hailstone as the one used for comparison. A third rare type of growth, spongy growth, is not included; however Dennis and Musil (1973) did examine this type of growth. Spongy growth occurs due to excess water not being shed from a wet hailstone. This typically occurs when supercooled liquid is added to a growing hailstone so fast that it cannot completely freeze (Knight 1968). These hailstones can include liquid water into an outward developing ice lattice. Rogers and Yau (1996) noted that this liquid fraction may account for up to 20% of the hailstone. Once these hailstones encounter subfreezing cloud temperatures, this entrapped liquid will freeze if the hailstone's temperature falls below 0°C. HAILCAST does allow this by using the Dennis and Musil (1973) equations, which calculates hailstone temperature (or heat budget) to determine which type of growth is occurring.

Hailstones incur all three modes of growth as indicated by a hailstone's cross section (Figure 3). Knight and Knight (2005) stated "…the growth modes of hailstones are largely determined by this balance between accretion rate and heat transfer which is a function primarily of the hailstone fall velocity, the cloud temperature, and the cloud LWC". If accretion rate is less than the heat transfer rate, dry growth occurs, and if the reverse is true, then wet growth occurs. It is important to note that the hailstone temperature is assumed below freezing during growth, while during wet growth the hailstone temperature is at freezing.

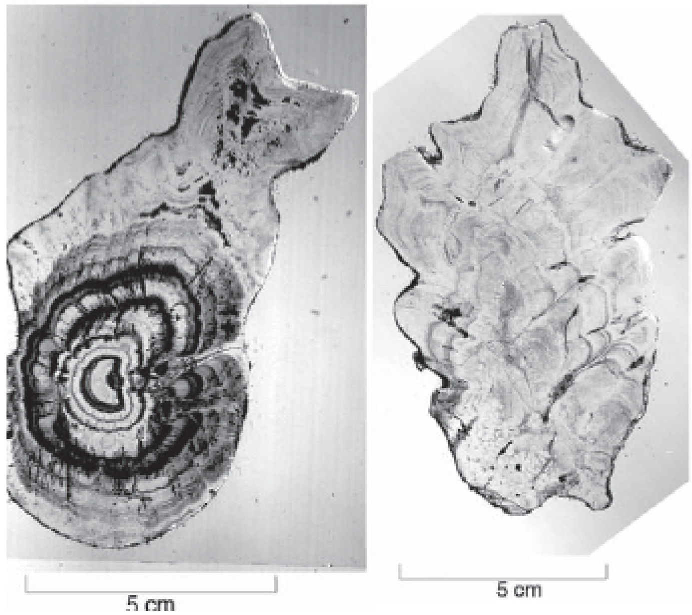

Figure 4. Cross Sections of Different Hailstones. One on left indicates spongy growth by large bubbles in center and one on the right grew by only wet growth. (After Knight and Knight 2005)

In his first article, Musil (1970) examined dry and wet hail growth in feeder clouds over the Great Plains. His 1-D model examined hail growth in regards to cloud water content and temperatures. It differed from many other hail models at that time by being time dependent. The complete derivation can be viewed in the article's appendix, so only the final equation will be stated here. For dry growth, the equation is

$$\frac{dR}{dt} = \frac{V_t X_l E_l}{4\rho_e} + \frac{V_t X_s E_s}{4\rho_e} \tag{2}$$

where V_t is the hailstone's terminal velocity, X_l is the liquid water content (LWC), E_l is the collection efficiency for liquid, ρ_e is the environment density, X_s is the ice crystal content, and E_s is the collection efficiency for ice. The first part on the RHS of the equation is growth due to the accretion of liquid water (cloud and rain droplets), while the second part is due to the accretion of ice particles. Terminal velocity depends on the embryo size. In his study he examined embryos sized from 20 µm to 50 µm. This was in line with measurements made during and before that time of large cloud droplets concentration near the base of the clouds. Researchers proposed that this was a primary

source region for hailstone embryos. As Orville (1977) stated graupel is also a main source of hail stone embryos. These typically are found in the colder regions of the U.S. Rasmussen and Heysmfield (1987a) and Heymsfield (1982) indicated that this graupel originated from the riming of aggregate crystals in cumulus towers flanking the mature cloud. Droplets shed from the surface of melting hailstones also could be a possible source for hailstone embryo. What is known is that the location of injection is more vital for rapid hail growth than beginning embryo size (Xu 1983). Although Musil started with 20-50µm embryos, this test model examines hail growth using four starting radii: 50µm, 100µm, 200µm, 300µm. HAILCAST begins with only one size, 300 µm. These were based on embryo sources mentioned above, as well as research completed by Brimelow (2002) and Macklin (1977). The embryo starting size ranges from a large cloud droplet to small graupel, thus covering some of the possible embryo sources for hailstone growth. Although Musil describe a method to approximate terminal velocity based on the embryo size, this study used equations from Rogers and Yau (1996) due to their simplicity. For embryos in the range of 40µm to .6mm, an approximate formula for terminal velocity is

$$V_t = k_3 r \tag{3}$$

where r is radius (mm) and $k_3 = 8\times10^3 s^{-1}$. For radii larger than .6mm, the formula is

$$V_t = \left(\frac{4g\rho_e D}{3\rho_a C_D}\right) \tag{4}$$

where g is gravity, C_D is the drag coefficient (.6), D is the hailstone diameter, ρ_e is particle density , and ρ_a is the air density. Therefore these are the only equation required for computation. These formulas do not take into account the affects of changing air density, where a droplet falls faster aloft than at sea level (Rogers and Yau 1996). Thus errors are possible using this approximation; however they are measured to be minor. Atmospheric density in the dry growth equation is calculated by using the equation of state

$$\rho_e = \frac{P}{R_a T_v} \tag{5}$$

where P is pressure in dynes cm^{-2}, T_v is virtual temperature in K, and R_a is gas constant for dry air (2.87×10^6 ergs gm^{-1}K^{-1}). Virtual temperature is calculated using the equation $T_v = T(1 + .61r)$ where T is the environment temperature and r is the water vapor mixing ratio. Hailstone density was kept constant at .9 gm·cm^{-3}, the same value used by Musil (1970), Dennis and Musil (1973), Takahashi (1976), Pino and Moore (1991), and Brimelow (2002). It appears that this value has become an acceptable value for hail density. Other researchers experimented with changing density values; however for this study the value will remain a constant.

The collection efficiency for ice and water was set at .25 and 1 respectively. This implies that hailstones sweep out all available water vapor and only a quarter of the ice. These values were also used by Musil (1970), Dennis and Musil (1973), Wisner et al. (1972), and Brimelow (2002). Errors are possible using these values since they assume a hailstone is a perfect sphere. In reality, hailstones differ from a near perfect sphere as it grows (Macklin 1977). Even though these values do not change, cloud water and cloud ice values do change. In this study, only up and down trajectories are examined, however in reality, hailstones will not descend exactly over the same path it ascended. As the Fleming storm shows (Figure 5), hail embryos have a wide array of possible paths or regions to move through. Some are more susceptible to growth than others. These differing paths are partly the result of shear which Das (1962) described. He noted that thunderstorms formed under strong vertical wind shear than without shear; however that did not translate into larger hail. It was the exact opposite, where less shear resulted in larger hail.

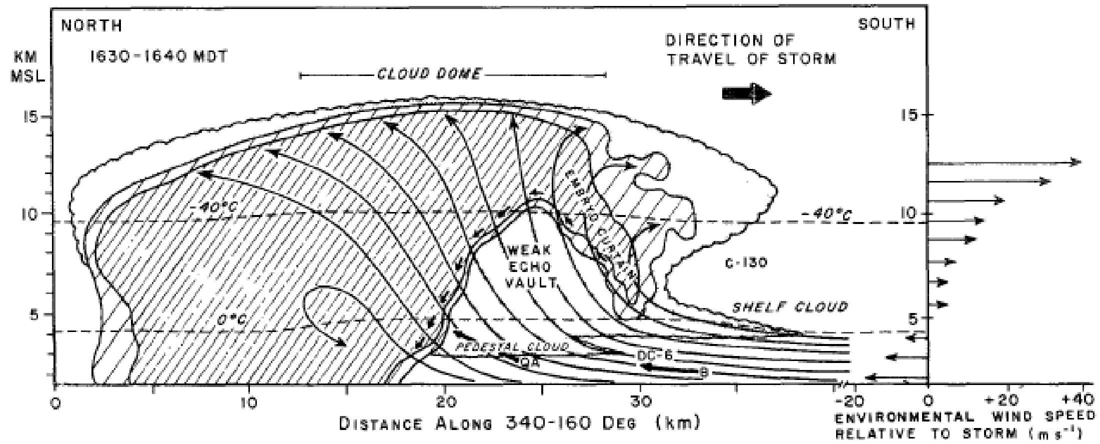

Figure 5. Vertical section of Fleming storm superimposed on the radar echo pattern. Short thin arrows represent one possible hailstone trajectory. (From Orville and Kopp 1977)

In addition hailstones are more likely to be blown out of the thunderstorms when shear is weak or nonexistent if the strong updrafts are located near the top of the cloud. Therefore a balance must be obtained between weak and strong shear to maximize hail growth or allow the hail embryo to spend the maximum amount of time in the greatest LWC region. This region is sometimes referred as the adiabatic liquid water core (Wallace and Hobbs 2004). It is very rare for a thunderstorm to obtain LWC values close to adiabatic. This study, however, is using only one vertical profile per event and to maximize hail growth, the largest LWC is required. Therefore the method to calculate the adiabatic LWC is the only one to use. Also, note that these profiles are from a 15 km model, and it is likely many of the profiles do not contain the cloud's maximum LWC. This could in the end limit the LWC; however the approach is suitable for the model resolution being used.

The last variables for the dry growth equation are the cloud water and ice content. The method being employed in this study for cloud water (X_l) is taken as the difference between a level's saturation mixing ratio and the saturation mixing ratio at the LCL, and multiplying this by the air density of the layer. Dennis and Musil (1973) used this method to calculate total water content, along with adjusting it downward during the earlier life of the cloud (entrainment). For ice content (X_s), the Musil (1970) approach

will also be used, which is based off the work of Valsi and Stansbury (1965). Musil (1970) defined a graph (Figure 6), in which the freezing process begins at -20˚C with completion by -40˚C. This study took the numbers from the graph and used a cubic interpolation to acquire the percent of liquid water frozen based on temperature at that level. This was accomplished after calculating the LWC at each level from the vertical profile.

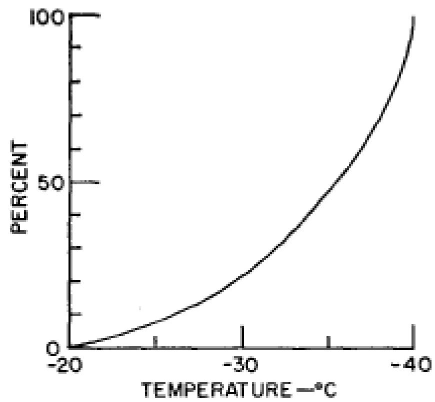

Figure 6. Assumed percentage of frozen cloud water as a function of in-cloud temperature (From Musil 1970)

As mentioned before the other type of growth that is examined is wet growth. Wet growth is defined as when hail growth occurs when the temperature of the hailstone is at 0°C. For these conditions, a hailstone is unable to dispel all the heat resulting from collisions with liquid droplets. Temperature of the hailstone begins to rise due to the release of latent heat of fusion by the accreted droplets (Macklin 1977) and hailstone growth is related to the rate at which heat can be transferred away from the storm to the environment. Typically this is a smaller growth method than dry, and would occur near the middle of the cloud where temperatures are closer to freezing (Xu 1983). The wet growth equation is as follows:

$$\frac{dR}{dt} = \left(\frac{1}{L_f + C_w T}\right)\left(\frac{a}{R\rho_e}\right)\left(-KT + LD\Delta\rho_w\right) + \left(\frac{V_t X_s E_s}{4\rho_e}\right)\left(1 - \frac{C_i T}{L_f + C_w T}\right) \tag{6}$$

where L_f is latent heat of fusion (79.7 calgm^{-1}), C_W is specific heat of liquid water [1.0 calgm^{-1}(°C)$^{-1}$], a is the ventilation coefficient (dimensionless), R is the hail radius, K is

thermal conductivity of cloud air [cal·gm^{-1}sec^{-1}(˚C)$^{-1}$], L is latent heat of vaporization (597.3 cal·gm^{-1}), D is diffusivity of water vapor (cm^{2}sec^{-1}), $\Delta\rho_w$ is the saturation vapor density at the temperature of the hailstone temperature minus that at the temperature of the cloud air (gm·cm^{-3}), C_i is the specific heat of ice [.5 cal·gm^{-1}(˚C)$^{-1}$], and the rest of the variables or constants are the same as in the dry growth equations. For wet growth, collection efficiency is assumed to be 100% or one. Instead of using the ventilation coefficient equations described by Musil, this study used a simpler equation noted by Rogers and Yau (1996). They used the equation $f = 1.00 + .09R_e$ for $0 \le R_e \le 2.5$ or $78 + .28R_e^{.5}$ for $R_e \ge 2.5$ where R_e is the Reynolds number of the flow around the drop defined as

$$R_e = \frac{2RV_t\rho_e}{\eta} \tag{7}$$

in which η is the viscosity of the air (1gm·cm^{-1}sec^{-1}). The viscosity of the air is determined by the equation $\eta = (1.718 + .0052T)10^{-4}$. Diffusivity is given by the equation

$$D = D_0 \left(\frac{T}{T_0}\right)^n \frac{P}{P_0} \tag{8}$$

where P_0 is 1000 mb, T_0 is 273.2 K, n is an empirical constant (1.81), and D_0 is 2.21×10^{-5} m^{2}s^{-1} (Rogers and Yau 1996). Conductivity (K) is calculated by the equation $K = (5.8 + .0184T)10^{-5}$ where temperature is in ˚C. The temperature in the final equation is actually the difference between hailstone and the environment. Instead, this study follows Musil (1970) in assuming that the hailstone temperature remains 0˚C. Dennis and Musil (1973) treated the hailstone temperature as a variable, allowing it to rise and fall based on the heat exchanges. This was simulating spongy growth. They discovered spongy growth actually tended to reduce the overall size. Thus using just wet and dry growth may result in higher than actual hail size, especially if melting is ignored. This probably will be offset by the multiple assumptions of the other variables, including the $\Delta\rho_w$ term. As mentioned before this term is the difference in saturation vapor density

between the hailstone and the environment and is difficult to calculate without making one assumption. In this study, hail is treated as ice and the environment (or cloud air) is assumed to be water. Then using the equation of state for an ideal gas and separating it into two equations:

$$e_i = \rho_{v(i)} R_v T_i \tag{9}$$

$$e_w = \rho_{v(w)} R_v T_w \tag{10}$$

where e is the saturation vapor pressure, ρ_v is saturation vapor density, and R_v is the gas constant. Solving for saturation vapor density (ρ_v) in each equation, and taking the difference between the two, results in

$$\rho_{v(i)} - \rho_{v(w)} = \frac{1}{R_v T_e}\left[e_i - e_w\right]. \tag{11}$$

Since the hailstone and environment temperatures are equal, T_i and T_w are also equal. Both temperatures are replaced by T_e. In order to calculate the differences between saturation vapor pressures, this study used two simple equations from Rogers and Yau (1996). For ice, an approximation is $e_s = Ae^{-B/T}$ where A equals 3.41×10^9 kPa and B equals 6.13×10^3 K. The same equation is used for water, however A is know 2.13×10^8 kPa and B is 5.42×10^3 K. Xu (1983) used a similar approach in calculating the difference in saturation vapor density. Based the values of A and B for ice and water, this overall term will be small but positive near the freezing level.

All of these variables contained in the wet growth equations contributed in a positive or negative way towards hail growth. Unlike the main ingredients, their effect is minor. Diffusivity or the rate of diffusion is where the water vapor molecules meander through the air (Stull 1995) from moist to drier air. A large gradient results in droplets growing faster whereby the lower humidity near the droplets are the results of growing droplets removing water vapor by condensation from adjacent air. The ventilation

coefficients are based on aerodynamic theory (Rogers and Yau 1996). As Rogers and Yau (1996) noted, the coefficient equals zero for a droplet at rest and increases with increasing fall speed. As the fall speed increases, the vapor field surrounding it becomes distorted. To account for these effects, a ventilation coefficient is added to the equations for latent heat and mass increases. Overall the effect is negligible for cloud droplets, but can be very important or significant for hail or rain drops. Conduction is simply based on the temperature of the environment. It varies from 2.63×10^{-2} to 2.07×10^{-2} for temperature range of 30˚C to -40˚C (Rogers and Yau 1996). The same can be said about the viscosity of the air. It also depends on the temperature and varies from 1.862×10^{-5} to 1.512×10^{-5} kg·m^{-1}s^{-1} for range of temperatures from 30˚C to -40˚C (Rogers and Yau 1996). Diffusivity, viscosity, and conductivity are all related to each other to a good approximation.

As stated previously, HAILCAST uses Dennis and Musil (1973) hailstone heat and mass budget equations to calculate hailstone temperature and fractional water content rates. It uses these rates to determine whether the hailstone is in a wet or dry growth region or critical breakup diameter. These equations arise from the heat exchanges between the hailstone and the environment, which includes conduction, evaporation/sublimation, accretion, and collection. A complete derivation of the change of rate of hailstone temperature is available in Appendix A of Dennis and Musil (1973), therefore this thesis states the final equation only, which is

$$\frac{dT_s}{dr} = -\frac{T_s}{M}\frac{dM}{dt} + \frac{1}{MC_i}\left[2\pi Da\left(KT - KT_s - L_s D_i \Delta\rho_w\right) + \frac{dM_w}{dt}(L_f + C_w T) + \frac{dM_i}{dt}C_i T\right] \quad (12)$$

where T_s is the hailstone temperature, $\frac{dM}{dt}$ is the change in mass (M) per unit time of a growing hailstone, D is hailstone diameter, $\frac{dM_w}{dt}$ is the change in mass per unit time due to collection of liquid droplets, $\frac{dM_i}{dt}$ is the change in mass per unit time due to

collection of ice crystals, and T is the ambient temperature. The remaining variables or constants are the same as in Musil (1970) wet and dry growth equations. Dennis and Musil (1973) derived this equation from the mass exchange equation

$$\frac{dM}{dt} = \frac{dM_w}{dt} + \frac{dM_i}{dt} \tag{14}$$

where $\frac{dM_w}{dt} = \frac{\pi D^2}{4} V_t X_w E_w$ and $\frac{dM_i}{dt} = \frac{\pi D^2}{4} V_t X_i E_i$. These equations are similar to Musil (1970) dry growth equations, however in HAILCAST hailstone density is not treated as a constant. It varies hailstone density from .9 to 1.0 gm cm^{-3} depending upon the fraction of liquid water contained within it. Fraction of liquid water (F_w) is calculated by the equation

$$\frac{dF_w}{dt} = -\frac{F_w}{M}\frac{dM}{dt} + \frac{1}{MC_i}\left[2\pi Da\left(KT - KT_s - L_s D_i \Delta\rho_w\right) + \frac{dM_w}{dt}(L_f + C_w T) + \frac{dM_i}{dt} C_i T\right]. \tag{15}$$

The complete derivation of the equation is once again in Dennis and Musil (1973) Appendix A. F_w is also important in the breakup of the hailstone (spongy) due to the buildup of liquid water. In the test model, all excess liquid water was assumed to be shedded. This is not always the case. Dennis and Musil (1973) and HAILCAST used an experimental relationship between fractional water content and critical diameter to determine breakup diameter. All of the ice and half of the water remain with the hailstone when using this breakup mechanism (Dennis and Musil 1973).

In summary, both growth modes (dry and wet) calculate hail growth over a pre-determine time period as long as the parcel temperature is below 0°C. Hailstone's terminal velocity (V_t) is determined by Eq. (3) and Eq. (4) depending on the hailstone's size. For dry hail growth calculations, Eq. (2) is used for parcel temperatures below -20°C. For wet hail growth calculations Eq. (6-11) are used for parcel temperatures

below -20 °C. If parcel temperature is between -20 °C and 0 °C, hail growth is then calculated by only the first parts on the RHS of Eq. (2) and Eq. (3).

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IV. MODEL SETUPS

The test model incorporates model derived omega and calculated updrafts via the buoyancy method. Since the hail growth and melting equations were discussed in the previous sections, only the updraft calculations and other various cloud parameters will be stated here.

A. TEST MODEL UPDRAFT CALCULATIONS

As was stated before, severe hail events were taken from NWS SPC hail event reports for the Central and Southern Plains. These events were filtered in order to remain close to the forecast time. After a program collected the vertical profile for each event, the output file was loaded into an algorithm that calculated hail growth for four different starting embryo radii using model vertical velocity (omega) and updrafts based on the buoyancy. In order to calculate the buoyancy, the parcel temperature at each model level had to be found. LCL height was calculated using the simple equation $Z_{LCL} = a(T - T_d)$ where $a = .125\,\mathrm{km}\cdot(^{\circ}\mathrm{C})^{-1}$ and T and T_d are the temperature and dewpoint respectively taken at the surface (Stull 1996). The LCL temperature is calculated by the equation where (Stull 1996). Once the LCL is located, the next step is to calculate the saturated lapse rate, which varies from 4 K·km^{-1} near the ground to 6-7 K·km^{-1} in the middle of the troposphere. This study incorporated the equation

$$\Gamma_s = \Gamma_d \left(\frac{1 + \left(\frac{a \cdot r_s}{T} \right)}{1 + \left(\frac{b \cdot r_s}{T^2} \right)} \right) \tag{16}$$

where Γ_d is the dry lapse rate (9.8 K·km^{-1}) and r_s is the saturation mixing ration (Stull 1996). r_s is computed by the equation

$$r_s = \frac{\varepsilon \cdot e_s}{P - e_s} \tag{17}$$

where $\varepsilon = .622$ g(vapor)/g(dry air), e_s is the saturation vapor pressure and P is the pressure. Saturation vapor pressure is calculated by

$$e_s = e_0 \cdot \exp\left(\frac{L}{R_v}\left(\frac{1}{T_0} - \frac{1}{T}\right)\right) \tag{18}$$

where $e_0 = .611$ kPa, $R_v = 461$ JK^{-1}kg^{-1}, $T_0 = 273$ K, and $L = 2.5 \times 10^6$ Jkg^{-1} (slightly different value for over ice) (Stull 1996). After the LCL height is found, the next level above is used as the starting point. Interpolation was not used since the actual sounding temperature is required to calculate the difference between the parcel and environment temperature. This required the parcel and environment levels to match. At the LCL, the saturation vapor pressure was calculated using the LCL temperature. Once the height difference between the first level and the LCL height, and the saturation lapse rate were found, they were multiplied together in order to retrieve the parcel temperature change between the LCL and the first model level above the LCL. Subtracting this from the LCL temperature resulted in the parcel temperature at that next level. Then the process begins all over again with first finding e_s, then r_s, and finally Γ_s, followed by multiplying this and the height difference between model levels in order to calculate the temperature change. This sequence continued until the last model level was retrieved. Thus at each level two temperatures profiles are created. The next step was to calculate buoyancy using the equation

$$B = \frac{T - T'}{T'} \tag{19}$$

where T is the environment temperature and T' is the parcel temperature. Then to find the updraft at that level, the equation

$$U^2 = U_0^2 + 2g\int_{z_0}^{z} B(z)dz \tag{20}$$

is used where U_0 is the updraft at the previous level, g is gravity, and $B(z)$ is the buoyancy at that level z. First the difference of buoyancy between the two levels is found. Then after inserting the height difference, the updraft at that new level is

calculated by taking the square root of the equation. It is important to note the updraft at the LCL is assumed zero. Moore and Pino (1990) also assumed this. They noted Bluestein et al. (1988) made the same assumption when comparing measured vertical velocities obtained from analyzed soundings from a storm intercept crew and computed vertical velocities assuming the parcel's vertical velocities is near zero at the LCL. His results proved the assumption valid. Once the updraft profile was created, it was passed on to the hail algorithm.

B. HAILCAST UPDRAFT CALCULATIONS

HAILCAST updrafts calculations are similar to the test model, however more complex and physically more complete. The 1-D cloud model in HAILCAST is similar to Anthes (1977) in that it includes entrainment (reduces updraft strength)

C. TEST MODEL SETUP

The hail algorithm breaks up into four separate algorithms: two for updraft (dry and wet growth) and two for model vertical velocities (dry and wet growth). At the beginning of each level the hail's terminal velocity is checked against the updraft or model vertical velocity. If the updraft is stronger than the terminal velocity, the hailstone raises to the new height, which is based on the difference between the two previous mentioned parameters multiplied by the time step. If the updraft is smaller than the terminal velocity, the hail will fall based on the same procedure just describe for the ascending hailstone. At that next level, the new hailstone radius is calculated as long as the vertical profile or calculated parcel temperatures are below zero. This is not an issue at the start, since the initial hail embryo is not injected into the mature thunderstorm until the temperature is below 0˚C. If the temperature is above 0˚C at that new level, the hailstone radius remains unchanged, and is once again checked against that level's updraft or vertical velocity. Also the moisture variable does not change, even if the hailstone passes multiple times through the same level. In reality, this simple up and down trajectories is usually more complex, as was mentioned before in discussing the Fleming storm. Browning (1977) discusses these trajectories via mutlicell hailstorm near

Raymer in northeast Colorado. This hailstorm indicated that a hail embryo or stone can travel tens of kilometers horizontally. The Fleming storm also led to a proposed three stage process of hail growth (Browning 1977) that included possible hailstone trajectories. Figure 7 depicts trajectories 1, 2, and 3, which correspond to Stage 1, Stage 2, and Stage 3 respectively. Stage 1, or the embryo state, represents small particles grown in a region of weak updrafts on the fight flank of the main updraft. Stage 2 is where these particles continue into the core of the main updraft with size range of several millimeters. Finally Stage 3 represents where embryos grow into hailstones during a single up-and-down trajectory (with minor fluctuations).

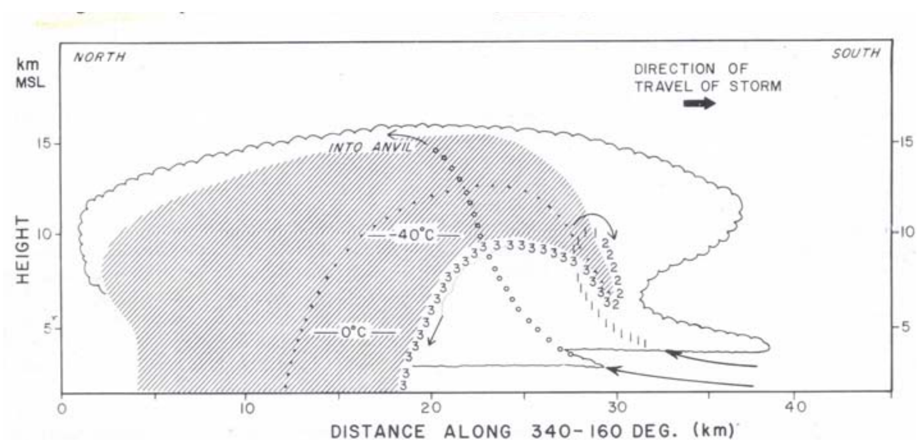

Figure 7. Vertical cross section of Figure 3. Trajectories 1, 2 and 3 represent the three stages in hail growth. Dotted trajectory represents a less favorable one for hail growth, while the circle trajectory will eject the particle out of the cloud top. (Adapted from Browning 1977)

The dotted trajectory represents less favored growth, in this case away from the edge of the vault, where the most intense updraft usually resides. The other trajectories (circles) indicate the possibility of embryos carried nearly out of the cloud top before they attained precipitation size or into the less favored regions for growth. In this thesis, focus will be on Stage 3. A similar growth stage theory based on the thunderstorm development was presented by Takahashi (1976). He broke down the hail growth into developing, mature, and dissipating stages (Figure 8). The developing stage will be ignored in this thesis since its assumes a mature or close to mature thunderstorm. The dissipating stage will be merged into the mature stage for this thesis since some large hail

is produced during the last stage. Unlike the last stage, this thesis will allow growth during ascent from the lower or middle part of the cloud. Although a mature thunderstorm is assumed, more than likely dissipating thunderstorms will be encountered which means downdrafts encompass at least the bottom half of the thunderstorm. This could cause some problems with the model. These potential problems will be discussed in further detail later.

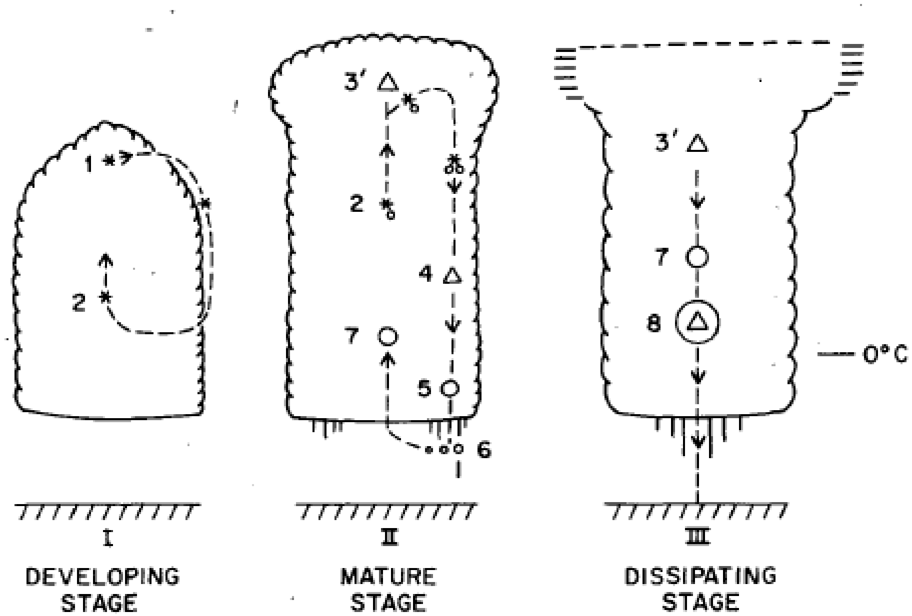

Figure 8. Hail formation process. Numbers 1 and 2 correspond to ice crystal recycling. Numbers 2 and 3 refer to recycled ice where the graupel forms, while graupel is falling near number 4. The graupel melts (5) and breakups (6), with some recycled (7) back into the updraft. In Stage III, the graupel falls, capturing large droplets (7) and finally forms into large hail (8). (Adapted Takahashi 1976)

Given that only one vertical profile is used for each event, the cloud will be assumed to be homogeneous in order to “simulate” a trajectory. It is very likely that many vertical profiles failed to fall in this favored area given the 15 km model resolution. Also it is very likely that the updraft slopes are tilted (Browning and Ludam 1960). This is the most idealized structure for an updraft so it can maintain itself continuously without interference. Typically it is tilted towards the up shear direction at low levels, turning in the down shear direction where it feeds the anvil. Unfortunately there is no way to verify

that the vertical profile from the MM5 contains the strongest updrafts, plus modeling these tilted updrafts is difficult and complex, and thus not examined in this thesis.

This up-and-down trajectory initially starts at the 0˚C level to ensure the embryo freezes. Most researchers injected the embryo at the LCL because they were examining the cloud growth as well as hail growth, however to ensure hail growth occurs from this model resolution, the freezing level was selected instead. Beyond the initial start, the hail embryo is free to move about. No growth is calculated when the temperature is above 0°C. Melting was not calculated in this thesis. Even though this subject will not be examined, its importance warrants a few notes about how it affects hail radius. Musil (1970) also did not directly address it in his paper, however Dennis and Musil (1973) did. Moore and Pino (1990) laid out a simple method for hailstone melting by taking the work of Mason (1956) and Macklin (1963, 1964) to solve for the change in hailstone temperature. From there they determined the time spent in above freezing layer before the hailstone hit the ground which results in a change in hailstone radius. This is similar to the one used in Dennis and Musil(1973) where they investigate spongy growth by looking first at the heat exchange rates. Rasmussen and Heymsfield (1987a, 1987b, 1987c) presented a much more advanced melting method, which Brimelow (2002) incorporated into his HAILCAST. This was a three point experiment examining the melting behavior of atmospheric ice. In the final article, Rasmussen and Heymsfield (1982c) noted that smaller particles melted more mass in unit times than larger hail. Essentially following the less complicated Macklin (1963) equations will give results similar to Rasmussen's one. Melting primarily depends on the radius of the hailstone, warming rate, and relative humidity in the downdraft. Figure 8 displays the improvement that Rasmussen and Heymsfield (1982c) experiments showed over an early experiment by Mason (1956). An earlier method allowed for quick estimate of whether or not melting would be a factor. Fawbush and Miller (1953) found that the hailstone will be unchanged if the wet bulb freezing level is less than 11,000 feet, or that the hailstone radius remains unchanged for nearly 9000 ft of free fall. Their work inspired Mason to explore the melting and develop physical equations previously mentioned. Although Musil (1970) did not directly address melting, he did state that only hailstones of .7 cm at

the freezing level would be allowed to reach the ground. This study will not address or examine melting. This approach may produce larger than expected sizes, however as was stated before the primary effects small hailstones, and since these hail sizes are from severe hail, this exclusion seems appropriate for this study.

This algorithm assumes a homogenous cloud both in time and space. Thus the updraft or vertical velocity profile remains constant. This is not the case in a real thunderstorm. Musil (1970) and Dennis and Musil (1973) examined changing updraft profile in their respective study. Musil (1970) used eight profiles, varying in peak of updraft and time of profile. The longest profile was 28 minutes with a peak of 25 kts. It is interesting to note that the three longest profiles were identical in strength below the maximum. Therefore the cloud was assumed to be mature. Although this method would be good to use, it is impractical due to the study's model resolution. If the study used a smaller resolution, it would be possible to include different times in order to create a multiple vertical velocity profiles. Dennis and Musil (1973) took it a step further in creating their vertical velocity profiles. They used the fist two terms of a Fourier series for their model's calculation of the updraft. This allowed them to create an unlimited amount of profiles depending on the cloud life, maximum cloud top, amplitude of the two harmonics, and lag time for the onset of the second harmonic. Once again if additional model profiles were available, this would definitely improve this algorithm. These changing profiles also applied the LWC, where Musil used eight different profiles of LWC in his feeder cloud model. As the time of the profile increased, the LWC increased to a maximum of 5 gm^{-3}. Like the vertical velocity profile, the three longest profiles were identical, obtaining moist adiabatic values. The earlier profiles only used 50-90% of the adiabatic values.

During a thunderstorm's life, placement of the updrafts is always changing. This test model assumes the updraft placement is fixed. Other studies did try to model the changing behavior of the updrafts. As mentioned above Dennis and Musil (1973) used their Fourier program to generate an updraft profile. This was based on other observations that early in a cloud's life, the maximum updraft persists mainly in the middle part of the cloud, while later the maximum updraft begins to move to the upper

part of the cloud. Also during this time, downdrafts began to appear in the lower part of the cloud. Their calculated updrafts or vertical velocities ranged from 10 to nearly 60 m s^{-1}. It is very rare, statistically, to observe updrafts of this magnitude. Danielsen (1977) noted that a necessary but not sufficient condition for hail formation is vertical velocity at least 12 m s^{-1}. This is based on the assumption that hailstone smaller than a raindrop will behave like a raindrop and melt completely before hitting the ground. Large hailstones will have a minimal terminal velocity of 12 ms^{-1}. That requires hailstones 3 to 10 cm in diameter to acquire updrafts of 30 to 50 ms^{-1}. Parcel theory can produce similar results as noted by Browning and Foote (1976) when examining Chrishom and Renick (1972) modified parcel theory in Alberta, Canada thunderstorms. Thus parcel theory tends to overestimate updraft magnitude, which results in larger hailstones. The parcel theory does not include non-hydrostatic pressure gradients, which can diminish the updraft velocity. This was also noted by Jewell and Brimelow (2004). They stated that this parcel method fails to include or subtract the effect of water loading, entrainment, and shear have on updraft strength. This method typically does not take into account storm mode or longevity, as well as skipping the microphysical processes important in hail formation and growth. In other words, a sounding resulting in large CAPE will generate very large hail. Also Jewel and Brimelow (2004_ mentioned something called storm type. This means what type of storm is producing the hail (single cell, mutlicell, supercell, etc). Brimelow (2002) compared these different storm types in the U.S. and Canada in order to determine a method for calculating updraft duration in which he based it on CAPEXSHEAR (See HAILCAST setup for additional information). Obviously updraft duration increases as you move from a single cell thunderstorm to a supercell. Other authors have also broken down thunderstorms into these categories in order to estimate the possible range of vertical velocities.

Table 1. Updraft duration determined by the multiplication of Shear and CAPE. (From Brimelow 2002)

CAPE × SHEAR ($m^2\ s^{-3}$)	≤1	2	3	≥5
Updraft duration (min)	20	35	45	60

Renick and Maxwell (1977) did this when they examined thunderstorms over Alberta. They used the same 3 categories for thunderstorms, ranging from single cell storms (weak to moderate updrafts) to 30 to 50 ms^{-1} found in supercells. They also indicated duration based on the storm which aided in their impressive hail forecasting procedure (Fig). Although this study does not examine any pre-storm environment parameters, it is important to determine these factors prior to using this simple model. Although this study does not examine any pre-storm environment parameters, it is important to determine these factors prior to using this simple model.

In Musil's conclusion section he found that storm rotation, sloping updraft and strong wind shear are not essential items in hailstone formation. Therefore this study does not examine these areas. Dennis and Musil (1973) found in their conclusion that the hailstone diameter is principally determined by the strength of the maximum updraft encountered by the hailstone and the temperature at which it occurs. This statement echoes the one delivered by Renick and Maxwell (1977) when they studied hail fall in Alberta. This was previously mentioned in the introduction. They created an effective nomogram that related hail size to maximum vertical velocity to temperature at the maximum vertical velocity level (Fig 9). During testing of this nomogram, they found that it forecasted the correct hail size 63% of the time and 80% of the time within one size category. Ironically they used a forecasted model (Load Moist Adiabatic) to calculate a vertical velocity profile. They made diagnostic model reruns of upper air data in order to examine various outputs that could be useful in creating a nomogram. Their nomogram was successful despite avoiding hailstone melting and hail physics. Both papers indicated that the maximum updraft and its location placed an upper limit on the hail size. Dennis and Musil (1973) discovered that a vertical velocity maximum at

temperatures -10 to -20°C tended to under forecast hail size. If the temperature is higher than -10°C, the model tends to over forecast the size of hail, thus a correction was required.

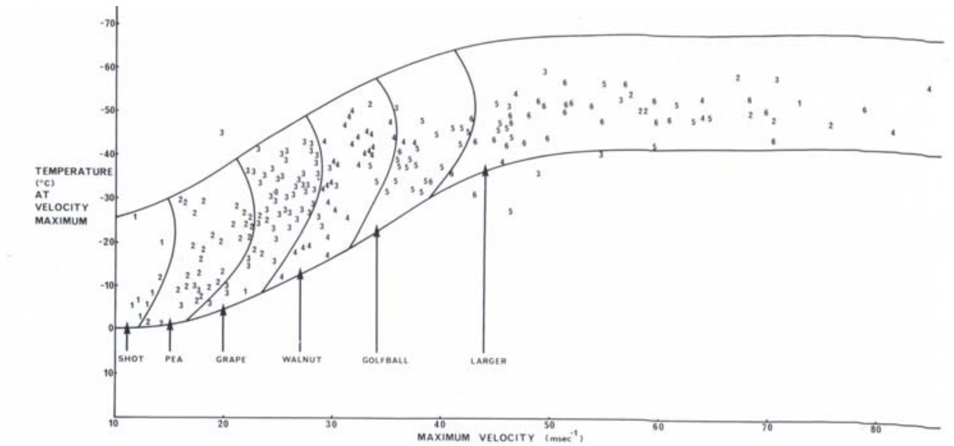

Figure 9. Hail Nomogram. Numbers correspond to the category. Hail categories are 1-6 which correspond to shot through larger hail size. (From Renick and Maxwell 1977)

In summary, the test model calculates the updraft using a simple parcel theory approach from model sounding, ignoring entrainment and water loading effects. Model vertical velocity profiles are converted from omegas to vertical velocities. These thermodynamic and model vertical velocity profiles remain fixed throughout the algorithm as it assumes the profiles are from mature thunderstorms. No interaction between hailstone and environment is allowed (thermodynamic parameters remain unchanged) and melting is ignored.

V. RESULTS

A. SAMPLE SET

This hail model algorithm required four separate program files to be called by the main program. In each of these separate programs, the algorithm runs through four separate beginning hail radii at four different vertical levels. Then dry and wet growth equations use model vertical velocities (converted from omega) and calculated updrafts via the parcel method to calculate 64 different ending hail sizes per each growth rate. Figures 10 and 11 display the maximum updrafts and vertical velocities versus the temperature at they occurs respectively. Note the huge differences between the two

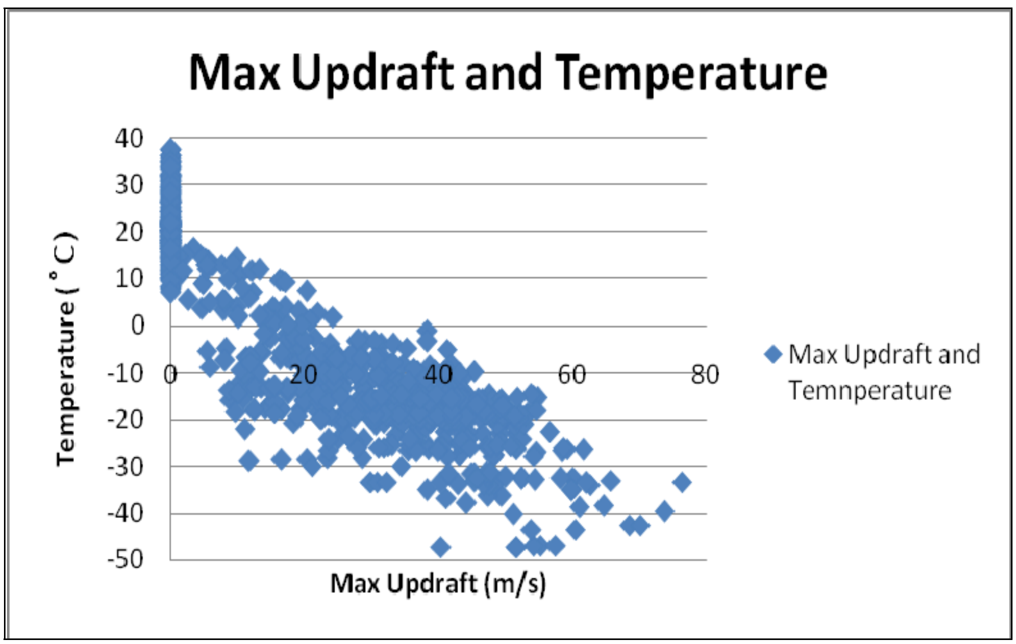

Figure 10. Max updraft and temperature at which it occurred for each hail event (804 total events).

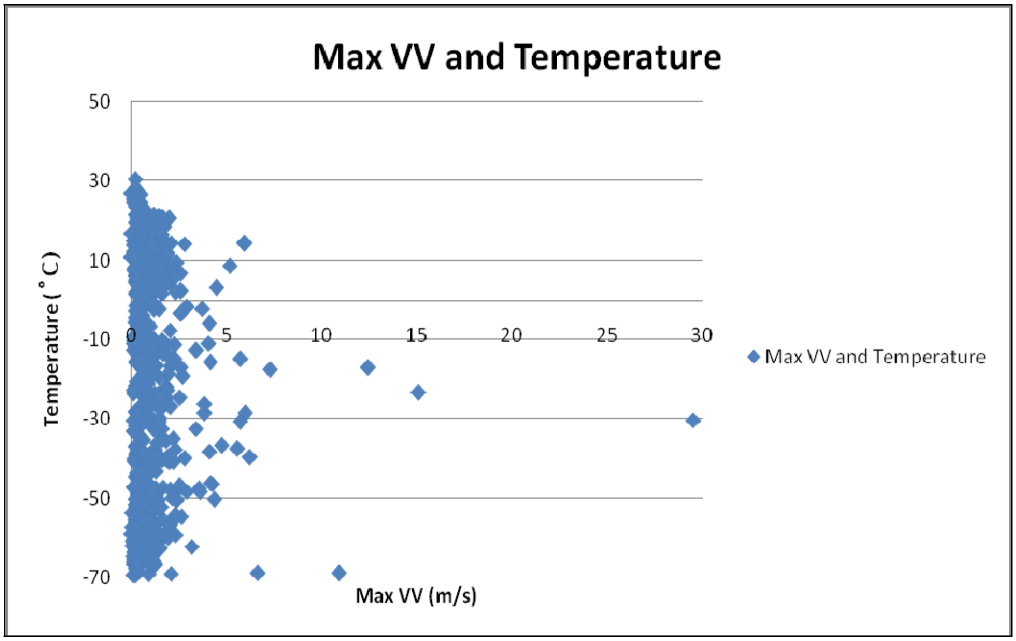

Figure 11. Max VV and temperature at which it occurred for each hail event.

graphs. Figure 10 shows a clear increase in updraft strength for decreasing temperature (T). This is due to the buoyancy not being depleted by any environmental interactions and lack of time dependence of the profile. The parcel accelerated upward gaining strength as T decrease. The max updrafts approach unrealistic value; however most of these are outside the hail formation layer and would not affect the hail growth. Figure 11 shows vertical velocities are generally under 10 m s^{-1}, with a few approaching 15 m s^{-1} in the prime hail formation zone. The lack of dependence on T shows that the model vertical motion is limited by various factors. Consequently maximum vertical velocity can occur at almost any level (T value).

In the first sample trial, the hailstone rose from the freezing level and grew until its terminal velocity was greater than the updraft or vertical velocity. There the hailstone would stop. This is unrealistic; however it allowed examination of some other important variables, such as the time step. Musil (1970) used a time step of four minutes with an integration time cap of 40 minutes, however since this thesis does not examine the full growth of the cloud, and therefore a shorter time step was chosen. HAILCAST used a

shorter time step of .5 seconds after a 60 second initial time step to start the process. After several trial runs, 150 seconds resulted in significant hail radius for nearly all the small sample set (~50 events). Larger time steps resulted in larger hailstones for the smaller beginning embryos; however the large starting embryos tended to go to zero. Thereafter the algorithm was adjusted to allow the hail to descend and ascend multiple times. This also required a time step adjustment downward to allow the embryo to remain in the cloud (array) for a longer time. The initial thought was that the updrafts would require a smaller time step than the vertical velocities due to magnitude differences between the two. The time step must be short enough to allow maximum time in the cloud, however not too short so that embryo stays unrealistic to long. In addition the model adapted HAILCAST's approach of beginning with a larger time step and then switching to a smaller one thereafter in order to give the beginning embryo an initial boost. For simplicity, the initial time step was exactly one-half of the follow-on steps. There is no physical reason for this and no testing was completed on different initial/follow-on time ratio. After testing several time steps for each four growth modes, the time steps were set at 60 and 30 seconds, 30 and 15 seconds, and 20 and 5 seconds for dry/wet growth using updrafts, wet growth using vertical velocities, and dry growth using vertical velocities respectively. The initial number is the first step, while the second is the time step used thereafter. Smaller time steps for growth modes using the updrafts resulted in smaller hailstones, while larger time steps resulted in very large hailstones or no hailstones at all. The same thing occurred for wet and dry growth using updrafts, however due to updrafts greater magnitude than vertical velocities; many of the hailstone sizes were unrealistic.

Once the time steps were set, the model ran through all 804 hail events. Although there was a possible 64 hailstone radii, no event resulted in this. Only 54 events resulted in 32 hailstone sizes, while the majority (250) of events resulted in 18 events. In order to compare the events sizes to the computed ones, the calculated hailstone were average from 16, 32, or 64 hailstone radii and placed in size bins centered on the standard reported hailstone sizes. Since the majority of the events were less than 2.25 inches, only eight bins were used. Any hail size greater than 2.25 was placed in the eight bin and any

calculated hail sizes less than .68 inches was ignored. Table 2 displays the results for eight different computed hailstone size averages. It's imported to note that bin one includes hailstones less than .75 inches. This was done to offset observation errors in reported hail sizes. Since hailstones are typically reported by a object of similar size, the actual hailstone size may vary from the standard size. Therefore calculated hailstone sizes greater than .68 inches were included in bin one.

Table 2. Number of hailstone in each size bin.

CAT[a]	Size	Observed	DM[b]	DU[b]	D[c]	WM[b]	WU[b]	W[c]	M[c]	U[c]	Overall[d]
1	0.75	192	43	75	62	1	12	17	0	111	163
2	0.88	175	145	53	110	1	15	11	4	100	65
3	1	189	188	35	108	0	16	3	2	71	32
4	1.25	36	253	113	251	0	13	15	1	73	24
5	1.5	19	106	170	179	1	21	7	2	46	24
6	1.75	141	2	110	29	0	8	6	0	15	12
7	2	18	0	46	21	0	2	6	0	15	4
8	2.25	38	0	35	32	0	51	18	0	53	14

[a] Size bins are .68 to .81, .82 to .94, .95 to 1.10, 1.11 to 1.34, 1.35 to 1.64, 1.65 to 1.89, 1.90 to 2.14, and greater than 2.14.

[b] DM (dry growth using vertical velocities), DU (dry growth using updrafts), WM (wet growth using vertical velocities), WU (wet growth using updrafts) are the average of 16 hailstone radii per event.

[c] D (dry growth averaging updrafts and vertical velocities together), W (wet growth averaging updrafts and vertical velocities together), M (average of vertical velocities from wet and dry growth), and U (average of updrafts from wet and dry growth) are the average of 32 hailstone radii per event.

[d] Overall (averaging both wet and dry growth using both updrafts and vertical velocities) is average of all 64 possible hailstone radii per event.

The majority of these reported calculated hailstones fell within the first three size bins. Only the model using updrafts produced large hailstone (bin 8), however far too many of them occurred in two columns. Wet growth using updrafts (WM) produced the most at 51. This number is deceiving since several hailstones reached unrealistic sizes (20+ inches) under this growth. Both wet growth (WM) and dry growth (DM) using

vertical velocities failed to generate any hail in bins 7 and 8. WM only produced three hailstones that were large enough to make the list. The results indicate that although the vertical velocities are strong enough to produce severe hail, they are unable to produce the large hailstones that occasionally occur over the region. This is to be expected since the model vertical velocities fail to include most of the convected component that is required to generate these strong updrafts to sustain the large hailstones. DM, however, did produced overall more severe hail than the rest. Table 3 further illustrates this point. DM generated 727 severe hailstones compared to only 661 for DU. Overall wet growth

Table 3. Breakdown of severe hail and hail/no hail for each average. Refer to Table 2 for acronyms and averaging technique for each column.

	DM	DU	D	WM	WU	W	M	U	Overall
GTE .75	727	661	776	4	131	74	19	436	276
GTE 0	804	803	804	574	690	772	804	803	804
% Svr Hail	90.4%	82.2%	96.5%	0.5%	16.3%	9.2%	2.4%	54.2%	34.3%
% Hail	100.0%	99.9%	100.0%	71.4%	85.8%	96.0%	100.0%	99.9%	100.0%

was unable to generate many severe hailstones. Since wet growth is a slower growth rate than dry, and typically occurs when cloud temperatures are under -9˚C, it should produce far less severe hailstones when it is a standalone model. This is less the case using updraft than vertical velocities since the updraft mode uses the parcel temperature instead of the sounding temperature (assumed to be the cloud temperatures) which is used by the vertical velocity model. This difference in temperatures could be large in some cases, resulting in different calculations for the multitude of parameters (especially for the difference in saturation vapor density between hailstone and the parcel). This, however, can also cause very large unrealistic hail sizes when using updrafts. Comparing updrafts versus the model vertical velocity numbers, updraft averaged (32 hailstone radii per event) 436 severe hailstones compare 19 for vertical velocities. This is mainly due to averaging since WM produce very few severe hailstones compared to WU or DU. This also effects the overall average (64 possible radii per event), resulting in low numbers of severe hailstones, but a good spread of sizes. Overall, the numbers from Table 3 make sense, however one might expect DU severe hailstone numbers to be higher and DM

numbers to be lower, mirroring the difference between WM and WU. The huge difference between wet and dry growth is somewhat expected since the majority of wet growth occurs near zero or in the lower part of the cloud, where the hailstone spends the minimum amount of time.

Model vertical velocities proved that in dry hail growth they are capable of producing more severe hail than produced using the updrafts. This is also the case when comparing accuracy. Figures 12-17 display four computed hailstone averages, along with a perfect forecast and trend line. Due to low numbers, WM was not examined.

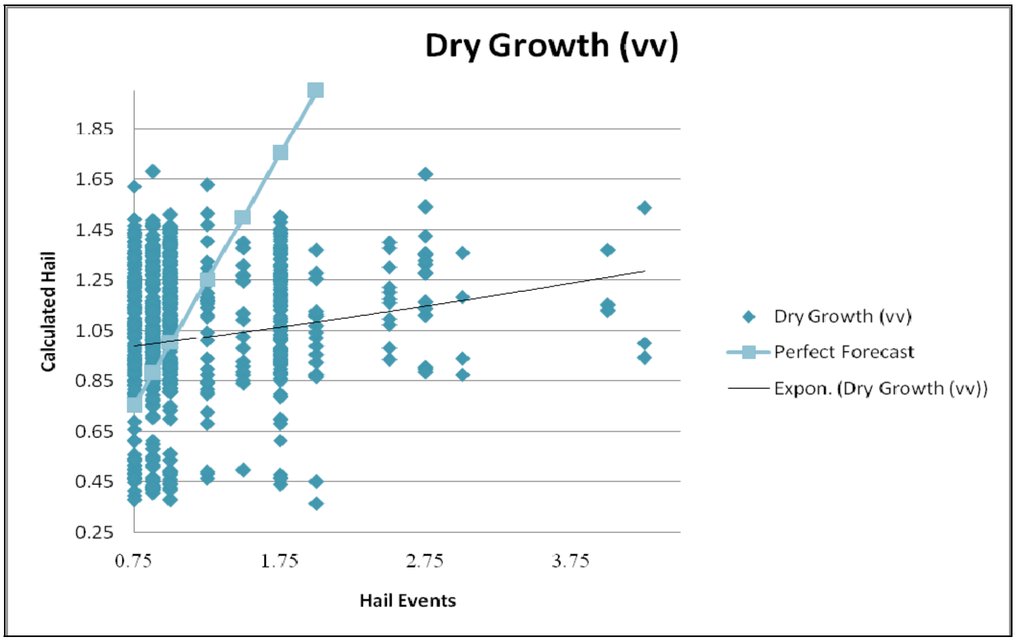

Figure 12. Dry growth using vertical velocities versus hail event sizes. Numbers are in inches. Calculated hailstone sizes are from an average of 16 hailstone radii. Trend lines and perfect forecast are also included.

Both DM and DU calculate a wide range of potential hail sizes for the smaller severe hail events, however the range narrows slightly as observed hail size increases. The range lifts up towards higher calculated hail sizes for DM, while for DU, the range pushed down towards the smaller calculated hail sizes. Take away the calculated hail sizes larger than 2”, the graph is very similar to an upside version of Figure 12. Although DU predicts large hail, it predicts it at the wrong time, or at smaller observed hail events. The maximum predicted hail size for each standard reported size also decreases as one

goes from smaller to larger observed hail sizes. This indicates that either the buoyancy is small or lacking in these cases. It is also possible the large hailstones were occurring in the mature stage of the thunderstorm (Figure 8) where downdrafts occupy most of the thunderstorm. Plus

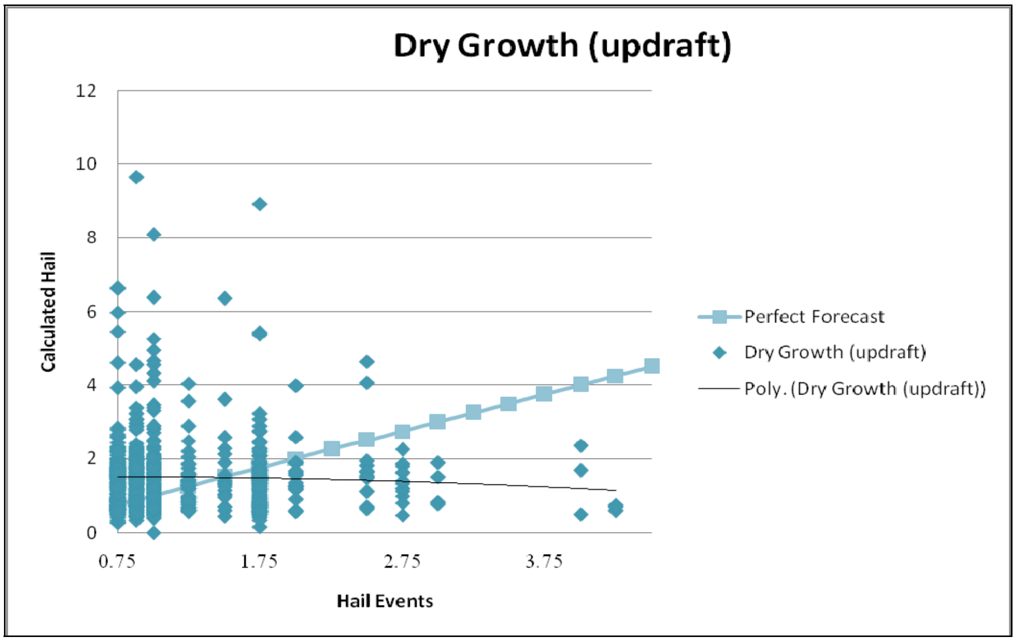

Figure 13. Dry growth using updrafts versus hail event sizes. Numbers are in inches. Calculated hailstone sizes are from an average of 16 hailstone radii per event.

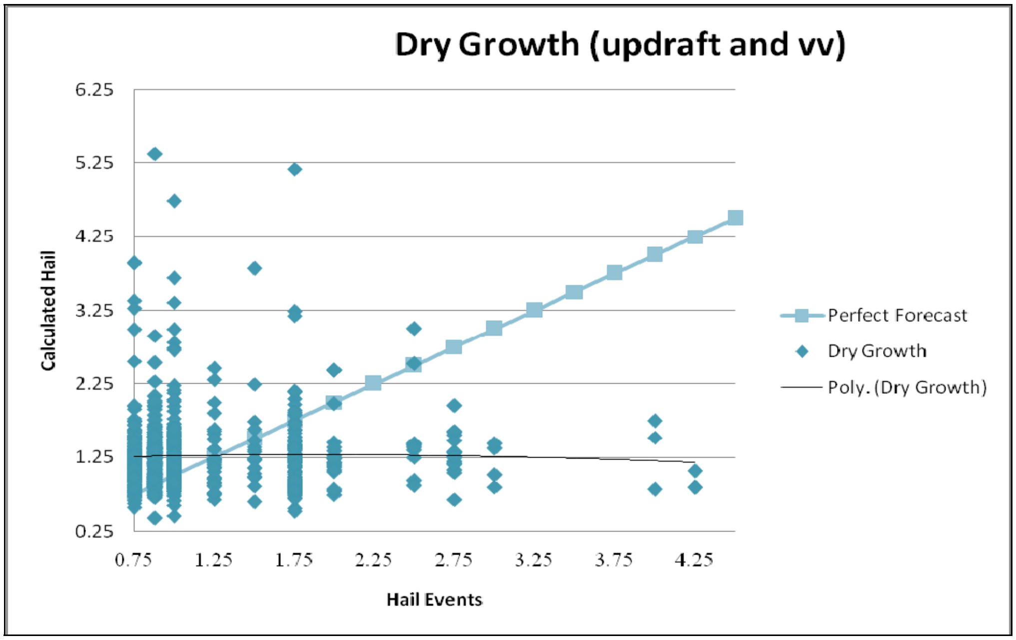

Figure 14. Dry growth (vertical velocity and updraft combined) versus hail event sizes. Numbers are in inches. Calculated hailstone sizes are from an average of 32 hailstone radii per event.

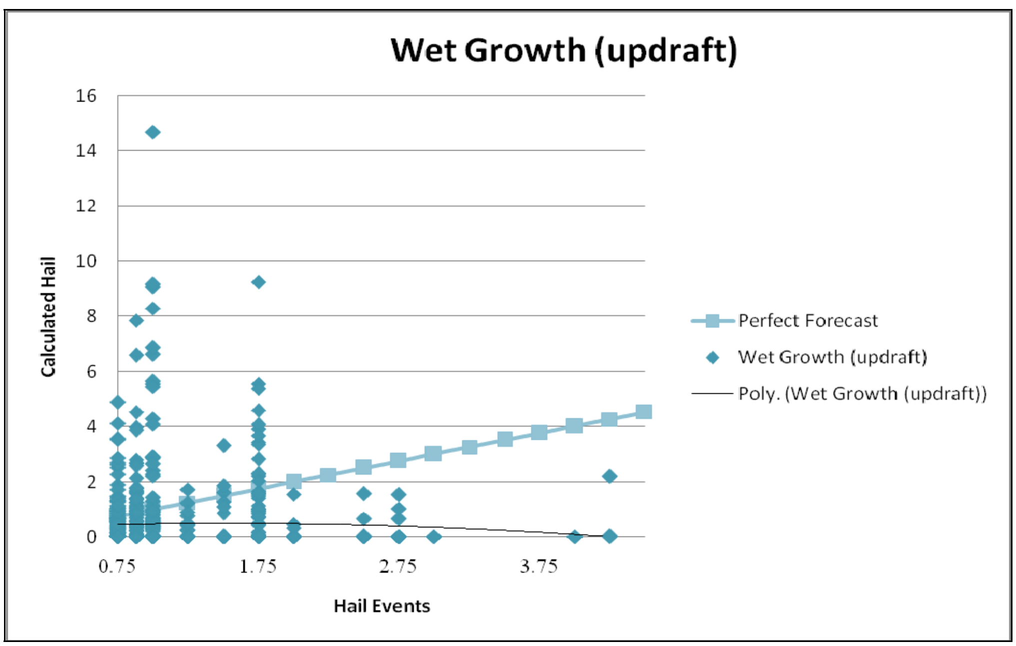

Figure 15. Wet Growth using updraft versus hail event size. Numbers are in inches. Calculated hailstone sizes are from an average of 16 hailstone radii per event.

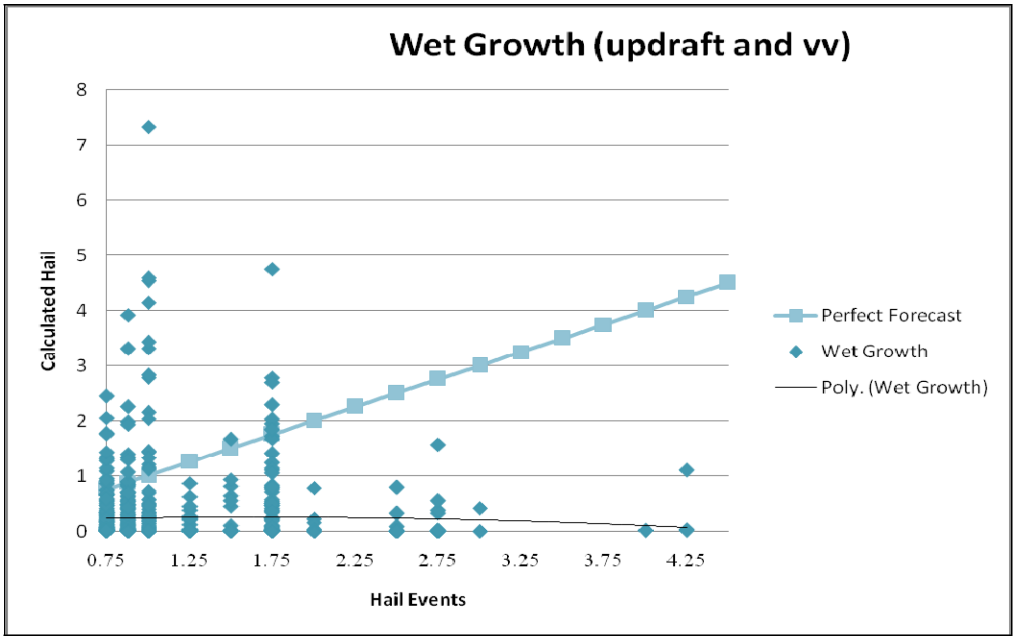

Figure 16. Wet Growth using updraft and vertical velocity versus hail event size. Numbers are in inches. Calculated hailstone sizes are from an average of 32 hailstone radii per event.

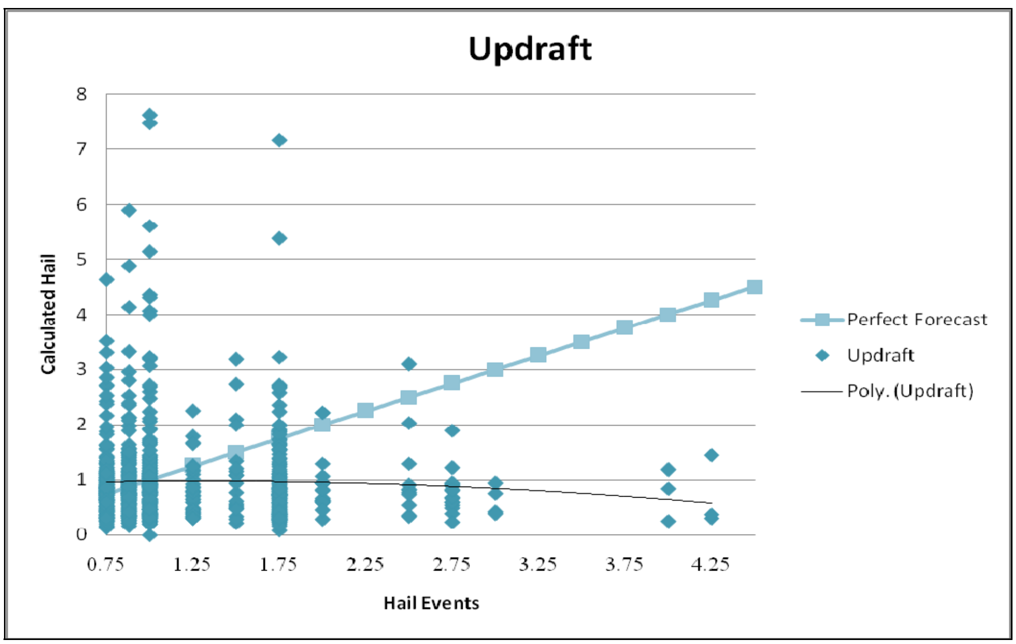

Figure 17. Updraft (dry and wet growth combined) versus hail event sizes. Numbers are in inches. Calculated hailstone sizes are from an average of 32 hailstone radii per event.

wet growth is likely responsible for the larger hailstones as they capture large droplets near the base of the cloud before falling out of the cloud. WU captures this possibility, albeit more frequent than the observed rate (39% compared to 5%). Plus the number of large generated hail occurred at the 5˚C level, with the -10˚C level coming in second. It possible that these extremely large hailstone that initial began at the 5˚C never froze or never became a hailstone (simply a large rain droplet). This seems more plausible since the beginning embryo size of 50μm produce most of these large hailstones.

Thus far the charts indicated the capability of using vertical velocities in hail forecasting. Table 4 further strengthens the vertical velocities case by examining the accuracy errors between the various hail growth modes. The average error for DU was

Table 4. Error Comparisons between various modes of growth. Numbers are in inches.

Error Comparisons (inches)									
	Dry			Wet			Overall		
	Max	Min	Average	Max	Min	Average	Max	Min	Average
VV	3.310	0.001	0.417	4.244	0.225	1.162	3.000	0.003	0.644
Updraft	8.778	0.000	0.788	13.670	0.009	1.189	6.616	0.000	0.688
Overall	4.495	0.002	0.527	6.337	0.006	1.086	3.817	0.001	0.576

nearly four-tenths of an inch greater than DM average error. Overall, errors were less for dry than wet growth, and slightly less for vertical velocities compared to updrafts. Errors were higher for wet growth using vertical velocities than those from wet growth using updrafts. WM generate very little sever hail, which resulted in a larger average error than WU, which did generate large hail, however too large in most cases. Even with the large possible number of ending hailstone radii, the average error was still poor. The same results occurred using HAILCAST (Jewell, personal communication) in that is nearly impossible to nail down a nearly perfect predicted hail size. Rather, a possible range of hailstone sizes is the best way to go.

Since DM is the most accurate of the growth modes, a further investigation is warranted to examine the different starting levels for DM. Tables 5 and 6 list the errors for DM at each of the four levels and four beginning radii. The 0˚C level contains the

lowest accuracy errors followed by the 5˚C level. On the other hand, the different beginning radii did not show any real differences in accuracy hail between then. This pattern (not listed)

Table 5. Error Comparisons between various modes of growth. Numbers are in inches.

Error Comparisons for DM (inches)								
	5˚C	0˚C	Minus 5˚C	Minus 10˚C	50µm	100µm	200µm	300µm
Max	4.172	3.545	2.535	2.305	3.456	3.445	3.315	3.190
Min	0.000	0.001	0.001	0.000	0.001	0.000	0.000	0.000
Average	0.73	0.460	0.991	1.221	0.415	0.409	0.411	0.401

also continued overall, but averaging around .525 inches accuracy error instead. Looking at the average size of the hailstone at each level, the size increases from .292 inches at 5˚C level to .811 inches at the -10˚C level. This increase in hail size did lower the maximum error, however increased the average error to nearly one inch. DU produced similar results (Table 6); however errors were much higher for three out of the four levels

Table 6. Error Comparisons between various modes of growth. Numbers are in inches.

Error Comparisons for DU (inches)				
	5˚C	0˚C	Minus 5˚C	Minus 10˚C
Max	13.090	33.270	22.740	4.365
Min	0.000	0.000	0.001	0.000
Average	0.948	1.261	1.507	0.915

Only at the -10˚C level did DM error exceed DU. The maximum error decreased significantly at this level, indicating that the model produces far less unrealistic hailstone sizes. It, however, produced larger hailstones than DM, which allowed the error to drop below one inch.

Although this error analysis is a descent look at the accuracy, operationally it makes more sense to compare the hail size bins between forecasted and observed hail. These size bins were mentioned before when comparing the actual hail numbers. Comparing the results for DU and for DM found that DM accurately predicted the size within three size bins 551 of 806 (69%) events compare to 430 (53%) for DU. This trend continued for within two size bins, where DM had 359 (45%) compared to 307 (38%) for DU. Finally, DM predicted the correct size bin 14% (110), while DU only predicted the correct size 10% of the time (79).

In summary, for four growth modes, DM produced the most (721) severe hailstones for the 804 observed hail events. However due to the small magnitude of the vertical velocities (Figure 11) compared to the magnitude of the calculated updrafts (Figure 10), DM, as well as WM, could not generate large hailstones (Cat. 7 and 8) that fell from some thunderstorms. Error analysis of generated versus observed hail diameter yielded interesting results. DM error at 0.411 was over a quarter of an inch less than DU (0.788). This trend continued when comparing overall model vertical velocities versus updrafts, however the difference in this case was only 0.04 inches. Breaking down DM and DU into different levels and starting embryo sizes (Tables 5 and 6) showed that starting sizes had no effect on hail diameter accuracy. However starting level did make a difference, with 0˚C the least amount of error at 0.460 for DM. DU produced the least error at -10˚C level, with 0.915. Finally, comparing the hail size bins between forecasted and observed hail for DM and DU growth modes only yielded similar results to the error analysis. DM consistently beat out DU when correctly predicting the size within three size bins, two size bins, and the correct size.

VI. CONCLUSIONS

This thesis examined the usefulness of vertical velocities in predicting hail sizes from forecasted soundings. It also briefly looked at the differences between starting embryo radii and initial injection levels. Overall the model indicated the possible improvements in using vertical velocities instead of relying solely on updrafts calculated by the parcel theory. Even though the updrafts were calculated from simple method (no entrainment or water loading effects), their overall sporadic nature in location and strength led to a lower number of severe hailstones than using model vertical velocities (348 versus 727 in dry growth conditions). The updraft method also generated some highly unrealistic values. The smoothness of the vertical velocity profiles led to a large number of severe hailstones in the dry growth mode, however all under 2 inches. In addition, average error for DU exceeded DM by almost a quarter of an inch less. This trend did continue for W and U comparison, albeit the difference was much smaller. Both modes (DM and DU) produced a wide range of possible calculated hailstone sizes for each hail event; however for DM this range shrunk and lifted to higher calculated hailstone sizes as the observed hail event diameter increased. For DU, the range shifted to lower calculated hail sizes, indicating that the larger hailstones were in a region of strong downdrafts and gain more growth near the cloud base where the large droplets resided.

This thesis just began to shed light on the usefulness of model vertical velocity. To further investigate this, a complete hail model is required. Therefore verifying these observations in HAILCAST is the logical choice. HAILCAST includes melting and spongy growth by calculating the change in hailstone's temperature in order to determine if the hailstone is in dry or wet growth conditions. Since wet growth failed to produce a significant number of severe hailstones, a hybrid approach may be necessary. That is, model vertical velocities are used, however parcel temperature is also used in order to accurate account for the change in saturation vapor pressure density differences between the stone and the cloud.

There are ways to improve this model without directly pulling out many of the internal MM5 model equations or variables involved in the parameterizations. They are:

- Add melting to the model. Moore and Pino (1990) melting method is a good first start, however incorporating the findings of Rasmussen and Heymsfield (1987a, 1987b, and 1987c) would be more complete. Xu (1983) also includes similar hail melting equations in his three dimensional cloud model

- Include additional vertical profiles nearby, thus allowing horizontal movement of the embryo or hailstone. This would allow the users or forecaster to observe the surrounding environment which could allow adjustments to the hail growth equation variables or the vertical profile.

- Add an additional vertical profile from at an earlier and later time than the event time. This allows the variable to not only change with height, but also with time. Simple trajectories equations from Xu (1983) would work perfectly for this setup.

- Increasing the model resolution without changing the algorithm (5 km or smaller) would lead to a more accurate picture of the hailstone environment. This could improve the results, or make them worse.

- Also adding results from additional operational models could narrow down the possible size ranges, and eventually create probability charts for specific hail ranges. Brimelow et al. (2006) briefly mentioned this idea.

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